EE330 Laplace Transforms

Transform Properties

	f(t)	F(s)
Linearity	$c_1 f_1(t) + c_2 f_2(t)$	$c_1 F_1(s) + c_2 F_2(s)$
Differentiation	$\frac{d}{dt} f(t)$	$sF(s) - f(0^-)$
Double differentiation	$\frac{d^2}{dt^2} f(t)$	$s^2 F(s) - s f(0^-) - f'(0^-)$
Integration	$\int_{0-}^{\infty} f(t)dt$	$\frac{1}{s} F(s)$
Time shift	$f(t-t_0)u(t-t_0), \quad t_0 > 0$	$e^{-st_0} F(s)$

Transform Pairs

	f(t)	F(s)
Unit impulse	$d(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	$t\,u(t)$	$\frac{1}{s^2}$
n[th] integral of an impulse	$\int \cdots \int d(t)$	$\frac{1}{s^n}$
Power of t	$\frac{t^{n-1}}{(n-1)!} u(t)$	$\frac{1}{s^n}$
Derivative of an impulse	$d'(t)$	s
n[th] derivative of an impulse	$d^{(n)}(t)$	s^n
Exponential	$e^{-at} u(t)$	$\frac{1}{s+a}$
t times exponential	$te^{-at} u(t)$	$\frac{1}{(s+a)^2}$
Sine	$\sin(wt)u(t)$	$\frac{w}{s^2+w^2}$
Cosine	$\cos(wt)u(t)$	$\frac{s}{s^2+w^2}$
Damped sine	$e^{-at}\sin(wt)u(t)$	$\frac{w}{(s+a)^2+w^2}$
Damped cosine	$e^{-at}\cos(wt)u(t)$	$\frac{s+a}{(s+a)^2+w^2}$
Damped sinusoid	$\sqrt{c^2+d^2}\ \ e^{-at}\cos(wt - \tan^{-1}(\frac{d}{c}))u(t)$	$\frac{c(s+a)+dv}{(s+a)^2+w^2}$