

Chart Analysis Enhanced with Mathcad®

Flexible general-purpose mathematics software can be customized for RF design work

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The graphic analysis used in RF circuit design has historically benefitted from the invention of the Smith® Chart [1, 2]. Now, with the advent of math packages such as Matlab®, Mathematica®, Mathcad® and others, further enhancements are possible. This article reviews classic design examples.

With the capabilities of packages such as Mathcad, a designer can improve productivity and enhance analysis capability with minimum effort. This article discusses the generation of the chart in Mathcad; drawing on the chart within Mathcad; and solving circuit design problems. In addition, Mathcad shows the capability of comparative analysis (for example, device scattering parameter comparisons), modeling parasitics in devices, extracting Q , device stability, mapping, use of the extended chart and other RF design capabilities. Many of these capabilities benefit directly from the ease in which the math package handles complex numbers, coupled with the ease of presenting output in graphic form.

Numerous discussions have occurred within the Mathcad collaboratory and other sites [3, 4] addressing chart construction and techniques. Mathematica is an excellent example of some of the possible RF design techniques [6]. Some of the earlier approaches were too complicated and impeded the real utility of the chart. Either the programming detail was long and inflexible, or the axis titles required in the graphics display mode were so numerous [5] that additional desired data to be displayed was lost in the maze. A return to the fundamental definition and construction of the chart discussed in this article streamlined the process.

Chart construction in Mathcad

Constructing the chart in Mathcad is a three-step process. It is illustrated in Appendix A, shown on page 56. First, we generate the “real contour circles.” Second, we generate the “single real axis.” Finally, we generate the “reactance contours.”

If the inductance and capacitance regions are treated as separate constructs, four labels are required for graphics output. Although the inductance and capacitance contours are mirror images, the separate constructions have been left intact for clarity. Minimizing the number of required labels keeps the display overhead down and provides space for the desired data to be plotted.

The chart is constructed in a rectilinear system instead of polar, therefore entry into the chart requires conversion from a polar coordinate system to a rectilinear system. Data entry could include circle locations for power gain, noise figure, stability, reflection coefficient circles that maximize output power and generalized feedback mapping contours.

The chart requires manipulation of the complex reflection coefficient G . The location of G will be given as a vector with magnitude and angle. In addition, for general applications, a complete description for the location of G should be provided in the form of a circular location, including a value for center and radius. All of these possibilities are readily handled in Mathcad with the following equations:

$$x = c + r \cos(\theta) \quad (1)$$

and

$$y = c + ir \sin(\theta) \quad (2)$$

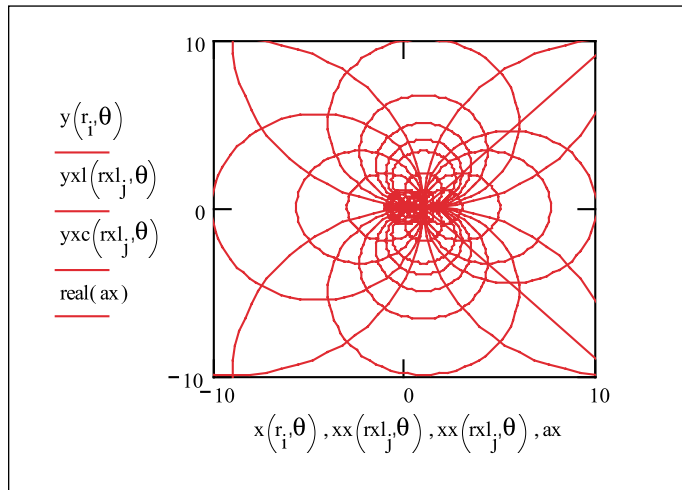
where i is the imaginary complex number, θ is a running variable from 0 to 2π radians, r is the radius or magnitude of the G circle or reflection coefficient, and c is the center location if a circle contour is required (this will be required for applications, such as gain plots, stability plane locations and noise contours). When the chart is plotted in Mathcad and no bounds are set, all four quadrants of circular contours are created. This extended chart is useful for studying designs with G greater than unity. The extended chart expands past the Smith chart unit circle and includes negative real resistance.

The case normally encountered in using the Smith chart for passive networks and non generative circuit cases contains only the unit circle and all the reactance contours terminate on this circle. The addition of control statements in Mathcad could constrain reactance contours so that they terminate on the unit circle. For this article, we have clipped the chart as required to contain whatever region may be of interest.

Extended chart

In Mathcad, the chart is simply bounded and “clipped” to unity, as illustrated in Appendix B on page 58. If negative resistance regions or stability analysis requires extending the chart, then the “clip” is extended or removed. In Figure 3, the boundary is extended to twice the unit circle radius.

Additional negative resistance contours are added by extending the R array table, as shown in Figure 4 and extending the “clip area.” The X array table is also increased for more detail and a popular sequence set is in a 1-2-5 format. Or, the table can be set to match any of the displays associated with a VNA such as the HP8753 series. In Figure 4, Γ is increased to 10 and additional negative resistance contours are exposed. If the same chart clips to $\Gamma = 1$, then the unit circle will be detailed.



▲ Figure 4. Extended Smith chart: $\Gamma = 10$.

Going around in circles

Circle plotting requires only the entry of center location and radius. For example, the location of constant VSWR contours requires a value of Γ that varies with radius proportional to an impedance, and a center at the origin of the chart. The equations and entry into Mathcad are shown below. This same concept is extended for computing and plotting noise contours, stability planes, feedback and n-port mapping, and evaluating component performance, for example Q . This is illustrated with the example in Appendix C that evaluates a device from the measured data section.

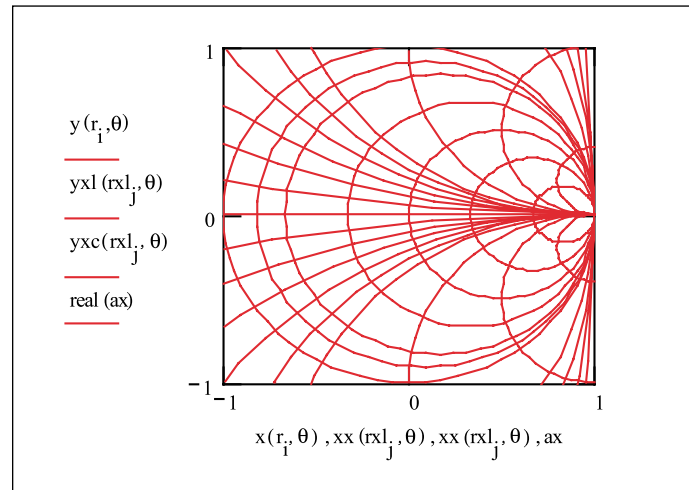
The SWR is assigned to an array table consisting of four fixed values. The Γ associated with each SWR value is calculated and the circle swept and x, y coordinated values entered into the chart. This construct could have been any type of contour. In the specific case of noise figure [7], the location of the circles would be geocentric with respect to F_{min} and the r_n of a device and at a specific angle set by the noise parameters associated with that device.

Evaluation of device from measured data

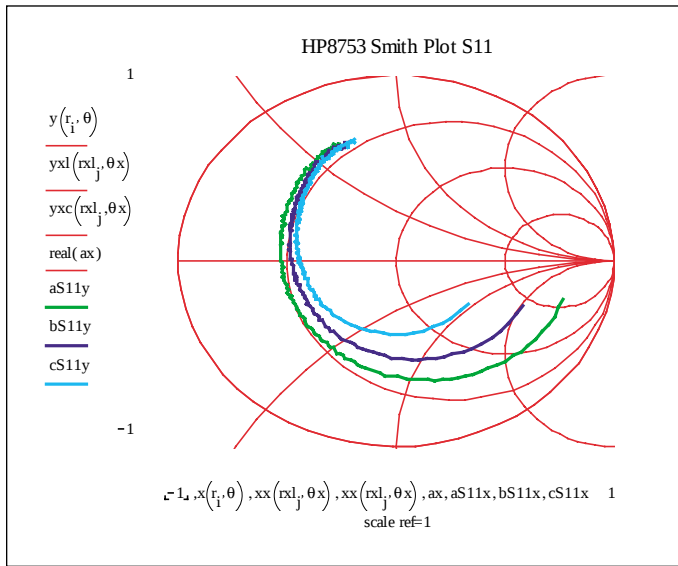
A set of Q contours are added to the chart by connecting all equal R-X contours, as shown in Appendix C on page 60. This would include the capacitive and inductive circles as they intersect the corresponding R values. Two circles are located with a radius of 1.414 and centers at normalized $R = 1$ and normalized $jX = +1$ and -1 [8]. With circular arcs drawn on the chart, this construct is used to evaluate component loaded Q .

The scattering data table is read from the VNA imported to the chart program and then plotted with the chart as an overlay. The component Q of a shunt configured-tuned inductor with parallel shunt parasitic capacitance is determined from the 3 dB bandwidth equation.

The “square” data points intersect the Q contours at the -3 dB power point, and f_0 is located as Γ crosses the real axis (diamond tick). The clockwise contour sweep is



▲ Figure 5. Unit Smith chart: $\Gamma = 1$.



▲ **Figure 9.** The result of increased collector current at constant collector voltage for a bipolar device over a 6 GHz frequency span.

indicative of a parallel resonance network. A mirror image of this contour exists for the dual, series resonant structure.

Scattering data is imported as a tabular array, plotted as a Γ coefficient, and markers added where the one port reflection coefficient intersects or cuts the Q -contour (shown in Appendix D on page 60). The markers are associated with a table of Γ as well as frequency. The Q is computed directly from the definition of $\Delta f/f_0$ within Mathcad and is illustrated in Figure 8. A data array is read from the VNA and consists of 200 data points. Either one port S -parameters for a shunt measured element or two port measurement of a series configured element may be used. The two port data is manipulated in Mathcad and converted to an equivalent set of one port reflection coefficient set and Γ plotted.

The center frequency data point is noted as the diamond tick in Figure 8 (shown in Appendix E on page 60), and the -3 dB power points are noted as square tick markers.

Comparative analysis

Scattering data is imported to Mathcad either via the serial port or a text file from a VNA. Other file formats such as CITI are also acceptable because they can be easily modified to a text file and placed into a “Touchstone format.” The previous example illustrated the evaluation of the Q of a passive component. Measured data from the VNA is imported to Mathcad, and using Smith chart construction techniques in Mathcad, unloaded Q is evaluated. Another powerful technique is the comparative analysis of scattering data plots for devices operating at different voltages or currents.

Comparative plots for devices during design and development are useful for gaining insight into device models and sensitivity. The read file command in Mathcad provides easy, simultaneous inspection of device parameter variations. Up to 12 different plots may be added to the chart and grouped for comparative analysis. For example, the following arrangement is used to compare input impedance and any parasitic resonance in a device as a function of current. Scattering data is directly read from the VNA and imported to construct routine and is displayed. The data array is 201 elements long and in standard Touchstone format, magnitude and angle.

$$end := rows(a)$$

$$rows(a) = 201$$

$$n := 1..end$$

$$aS11x_n := \left(a^{(2)} \right)_n \times \cos \left[\left[\left(a^{(3)} \right)_n \times \text{deg} \right] \right]$$

$$aS11y_n := \left[\left(a^{(2)} \right)_n \right] \times \sin \left[\left[\left(a^{(3)} \right)_n \times \text{deg} \right] \right]$$

Entry to the chart requires conversion to rectilinear coordinates. The plots below show the result of increased collector current at constant collector voltage for a bipolar device over a 6 GHz frequency span.

The increasing frequency sweep is clockwise. The device package shows a resonance below 6 GHz, and the low frequency performance (for example, the input impedance) is typical. An increase in r_e occurs as the total emitter current is decreased. With increasing frequency, the input impedance is dominated by the package parasitic, and at the end of the frequency sweep, no discernible difference is seen.

Conclusion

The Smith chart construct in circuit design and analysis is a powerful tool and graphics aid. Complementing this with a mathematics package such as Mathcad makes a desirable analytic and design tool. The designer can quickly verify and validate models and designs and can add additional mathematical processes to the system. This capability is not readily possible in many of the current RF simulation and design packages available. ■

References

1. P. Smith, “Transmission Line Calculator,” *Electronics*, Vol. 12, 1939 and 1944: 29–31.
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3. Mathcad® Collaboratory and Applications. Available online at www.mathsoft.com/appsindex.html and www.mathsoft.com.

4. Leonard M. Schwab, "The Smith Chart for Impedance Matching" (also see white paper, "The Smith Chart is Alive and Well in Cyber World"), *OnLine Symposium for Electronics Engineers* (archive at *Analog and RF Electronics*), October 24, 2000. Available online at www.osee.net.

5. H.J. Delgado and Michael H. Thursby, "Derivation of the Smith Chart Equations for use with Mathcad," *IEEE Antennas and Propagation Magazine*, Vol. 40, No. 2, April 1998: 99–101. Available online at http://ee.fit.edu/electrical/asl_page/journal.html.

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Vol. CT-13, No. 2, June 1966: 137–142.

8. Microwave Network Analyzer Applications, Hewlett Packard AN117-1, June 1970.

Author information

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APPENDIX A

R table

$i := 1..5 \quad \theta := 1..2 \times \pi$

Circle sweep for conductance and reactance contours, .01 radian step should give fine resolution.

$r_i :=$

0
.333
1
3

The table vector array describes r . Four values are used to generate the conductance circles at 150 ohms, 50 ohms, 16.67 ohms and the unit circle. The r table is input as a vector table and entries are made by inputting the first value followed by a comma for additional values.

$$(r, \theta) := \frac{r}{1+r} + \frac{1}{1+r} \times \cos(\theta)$$

x and y values generate the circles of conductance.

$$(r, \theta) := \frac{1}{1+r} \times \sin(\theta)$$

$ax := -1..1$
real (ax) : = 0

This sequence forms the real axis.

X table

$j := 1..3$

Same θ sweep used here, but a different angle sweep is permitted to achieve more or less resolution than the real axis.

$rxl_j :=$

$$xx(rxl, \theta) := 1 + \frac{1}{rxl} \times \cos(\theta)$$

$$yxl(rxl, \theta) := 1 + \frac{1}{rxl} + \frac{1}{rxl} \times \sin(\theta)$$

$$yxc(rxl, \theta) := 1 + \frac{-1}{rxl} + \frac{1}{rxl} \times \sin(\theta)$$

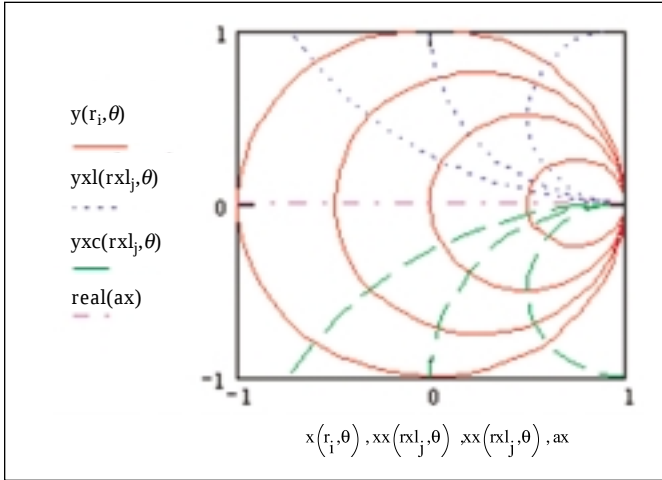
This set of sequences generates the X_c and X_L reactance circles. Note that $xx()$ is used for both contours and that the two other contours differ by a sign change. Again, rxl_j is a vector table loaded with values for three reactance contours: 100 ohms, 50 ohms, 25 ohms.

.5
1
2

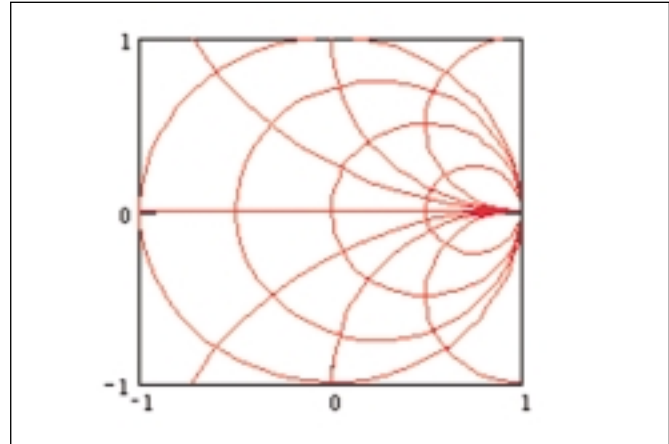
These tables can be extended and changed to add more detail to the chart. The R and X tables used here construct a chart that emulates the default chart used on the HP8753 series VNA. The chart provides for the inscription of a 3:1 VSWR circle.

continued on next page

The charts are limited in x and y dimension by clipping all values to +1 and -1.



▲ Figure 1. All four traces are shown.



▲ Figure 2. All traces are set to red; legends and labels are turned off.

APPENDIX B

Circles on the chart

Though the chart appears polar, the entry is in rectangular. The chart is sitting on a coordinate system bounded by $x = (-1, 0, +1)$ and $y = (-1, 0, +1)$. To place a circle anywhere on the unit circle or outside the unit circle to the extended chart, use the following:

$$\alpha := 0, .1..2 \times \pi$$

$$\beta := 1.2 \text{ radius}$$

$$\delta := 45 \text{ degrees center location}$$

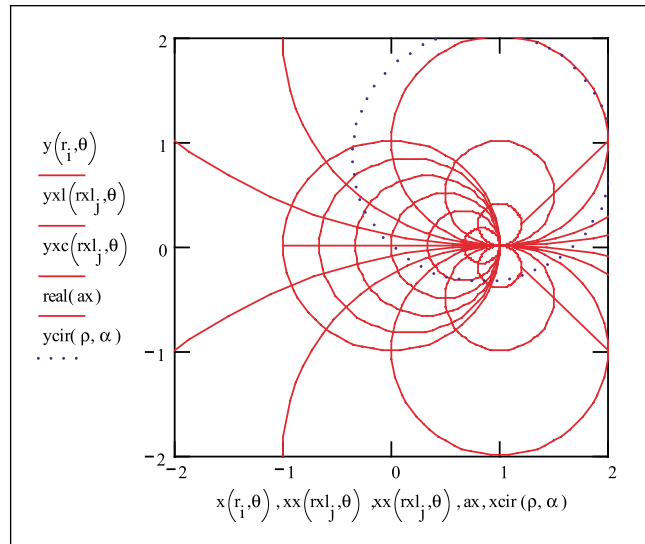
The values above could represent a stability circle location. Using an extended chart, the entire (un)stable locus is located and provides a technique for mapping feedback networks useful in oscillator design.

Convert the center location into a (x, y) pair.

$$xc := \beta \times \cos(\delta)$$

$$yc := \beta \times \sin(\delta)$$

Create the circle on the chart.



▲ Figure 3. Charting circles.

$$xcir(\beta, \alpha) := \beta \times \cos(\alpha) + xc$$

$$ycir(\beta, \alpha) := \beta \times \sin(\alpha) + yc$$

APPENDIX C

Select Circles or VSWR Plots:

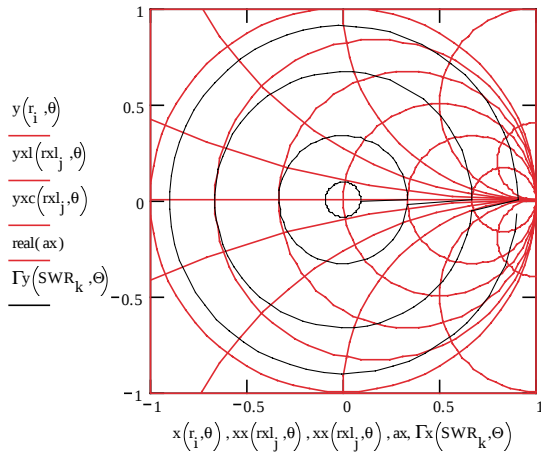
k := 1.. 4 := 0 .. 2

$$\Gamma_x(\text{SWR}, \theta) := \frac{\text{SWR} - 1}{\text{SWR} + 1} \cdot \cos(\theta) \quad y(\text{SWR}, \theta) := \frac{\text{SWR} - 1}{\text{SWR} + 1} \cdot \sin(\theta)$$

SWR_k :=

1.2
2
5
20

VSWR Contours



APPENDIX D

Generating Q circles for the chart

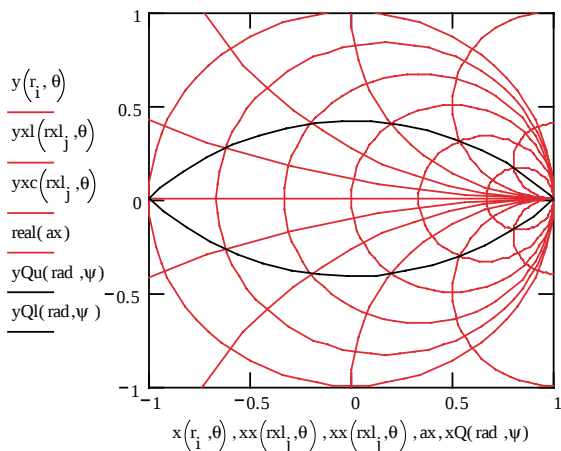
The Q contours intersect all points where $R = \pm jX_c$ or $\pm jX_L$. The center of such a circle is at $x = 0$, $y = 1$, and the radius is 1.414 or root (2).

$$\psi := 0 \text{ .. } 2 \quad \text{rad} := \sqrt{2} \quad \text{cent} := 1$$

$$xQ(\text{rad}, \psi) := \text{rad} \cdot \cos(\psi) \quad yQu(\text{rad}, \psi) := \text{rad} \cdot \sin(\psi) - 1$$

$$yQl(\text{rad}, \psi) := \text{rad} \cdot \sin(\psi) + 1$$

Smith Chart with Q contours



APPENDIX E

As a test of the Q chart, input a set of measured S-parameters with a given reflection coefficient Γ . f_o is 1.9 GHz and Q unloaded is to be determined.

Q := READPRN("a:QRESP.txt") nend := rows(Q) nend = 200 GHz := 1

q := 1.. nend

$$Qx_q := (Q^{<2>})_q \cdot \cos[(Q^{<3>})_q \cdot \text{deg}] \quad Qy_q := (Q^{<2>})_q \cdot \sin[(Q^{<3>})_q \cdot \text{deg}]$$

$$\text{mark1} := 48 \quad \text{freq1} := (Q^{<1>})_{\text{mark1}} \quad \text{freq1} = 1.799 \cdot \text{GHz}$$

$$\text{mark2} := 55 \quad \text{freq2} := (Q^{<1>})_{\text{mark2}} \quad \text{freq2} = 1.992 \cdot \text{GHz}$$

$$\text{markFo} := 52 \quad \text{freq3} := (Q^{<1>})_{\text{markFo}} \quad \text{freq3} = 1.91 \cdot \text{GHz}$$

$$Q := \frac{\text{freq3}}{\text{freq2} - \text{freq1}}$$

$$Q = 9.87$$

